

Tripotents in algebras: Invertibility and hyponormality

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Abstract

© 2014, Pleiades Publishing, Ltd. Let A be a unital algebra over complex field \mathbb{C} , I be the unit of A . An element $A \in A$ is called tripotent if $A^3 = A$. Let $A_{tri} = \{A \in A: A^3 = A\}$. We show that $A \in A_{tri}$ if and only if $I \pm A - A^2 \in A_{tri}$. We study invertibility properties of elements $I + \lambda A$ with $A \in A_{tri}$ and $\lambda \in \mathbb{C} \setminus \{-1, 1\}$. Let X be a Banach space with the approximation property and $A, B \in B(X)_{tri}$. If $A - B$ is a nuclear operator then $\text{tr}(A - B) \in \mathbb{C}$. We show that if H is a Hilbert space and an operator $A \in B(H)_{tri}$ is hyponormal or cohyponormal then $A = A^*$. We also prove that every $A \in B(H)_{tri}$ similar to a Hermitian tripotent.

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Keywords

algebra, Banach space, Hilbert space, hyponormal operator, idempotent, invertibility, nuclear operator, projection, similarity, symmetry, trace, tripotent